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EMPIRICAL STUDY OF CONDUIT RADIAL CROSS-SECTION DETERMINATION AND REPRESENTATION METHODS ON CAVERNOUS LIMESTONE POROSITY CHARACTERIZATION

Ira D. Sasowsky
Department of Geology and Environmental Science, The University of Akron, Akron OH 44325-4101 USA, ids@uakron.edu

Melisa R. Bishop
Department of Geology and Environmental Science, The University of Akron, Akron OH 44325-4101 USA

Radial cross sections are constructed during cave mapping in order to illustrate karst groundwater conduit (cave passage) morphology. These sections can also be employed in studies of porosity distribution and paleohydrology. Cave surveyors usually estimate left, right, up, and down (LRUD) distances from a survey station to the conduit wall, and these four values are used to construct the radial cross section, and occasionally integrated along the length of the passage to determine cave volume. This study evaluates the potential errors caused by LRUD estimation, as well as the effects of differing geometric approximations of passage shape. Passage dimensions at 18 stations of diverse size and morphology in Scott Hollow Cave, West Virginia were first estimated for LRUD and then precisely surveyed using a laser rangefinder taking 16 radial measurements. Results show that, depending upon the purpose of a survey, a reasonable approximation of passage shape might be made with fewer (four or eight) measurements. In cases where only four lengths are determined, approximation of the passage as an ellipse or rectangle provides a more accurate morphology and area than if portrayed as a quadrilateral. In the former case, average area errors were on the order of ±10%, as opposed to −45% in the latter. Surveyor estimates of LRUD give an average overestimate of 27%. Length errors compound, however, when areas are calculated. This results in an average cross-section area error (as quadrilateral) of 57% when using estimates instead of measurements. This may be problematic for such analyses as calculation of fluid storage volumes or paleodischarges.

Introduction

Accurate cave maps serve as the foundation for many studies of karst geology, hydrology, and biology, because most limestone cave passages are active or abandoned karst groundwater conduits. Refinement of survey and mapping techniques through the years and the proliferation of survey reduction software have made it easier to produce high quality representations of caves (Dasher, 1994). There are many different approaches to mapping (Dasher, 1994), but the usual strategy is to lay out a three-dimensional line of survey through the cave, referenced to a known point on the land surface. This establishes the humanly explorable length of the cave, position of the cave in space, and the topology of the conduits. In order to produce other than a simple line map of the cave, it is also necessary to determine passage width and height, or radial cross-section dimensions, at each survey station. To accomplish this, distances are recorded as left, right, up, and down (LRUD) values from the survey station to the cave wall, and thereby provide thickness to the spine created by the three dimensional survey line (Fig. 1a, b). These values can then be used to represent cave size/shape in the ways indicated in Table 1. In addition to allowing good representations of the cave, accurate radial cross sections are essential for many scientific purposes. They allow paleodischarge calculations when taken in conjunction with paleoflow indicators such as scallops (Curl, 1974), evaluation of cave origin and geologic controls (phreatic/vadose) when analyzed for morphology (*e.g.*, Palmer 1981, 1984; White, 1988), and determination of overall cave volume (and rock bulk porosity) when integrated along the length of the cave (Worthington *et al.*, 2000). It should be noted that in the case of volume and porosity, results are limited to discovered and proper (Curl 1964, 1966) caves. There will always be undiscovered or un-explorable porosity in the rock mass.

In spite of all of the above reasons to determine accurate radial cross sections, this aspect usually receives the least attention during a cave survey. In most cases the LRUD dimensions are not actually measured, but are estimated to the nearest whole foot or meter. This is done for the sake of expediency. Moreover, while there is a literature on the techniques of cave surveying and the sources of error therein (summarized in Dasher, 1994), scant attention has been paid to errors due to estimation of LRUD, or due to the approximations of passage cross-sectional shape based on only four distances.

In order to examine the quality of LRUD estimations and passage radial cross-section representations, we conducted precise radial surveys at a variety of stations in a cave (Fig. 1). We then compared the LRUD values estimated by surveyors to those that we measured. We also examined variation of cross-sectional area calculation caused by using different geometric approximations with the LRUD values, and by using additional radial measurements.

Study Location

Scott Hollow Cave (Monroe County, West Virginia; Fig. 2) contains 43.4 km (27.0 miles) (Gulden, 2006) of mapped passage, and was chosen for this study due to the wide variety of passage types present. Monroe County lies at the transition between the Appalachian Plateau and the Valley and Ridge geomorphic provinces, and the cave is developed within rocks of the Mississippian age Greenbrier Group and the underlying Maccrady Shale. The cave is situated in the western arm of the Sinks Grove Anticline (Davis, 1999), and contains passages of both phreatic and vadose origin, as well as modifications by breakdown, speleothem growth, and fluvial sedimentation.

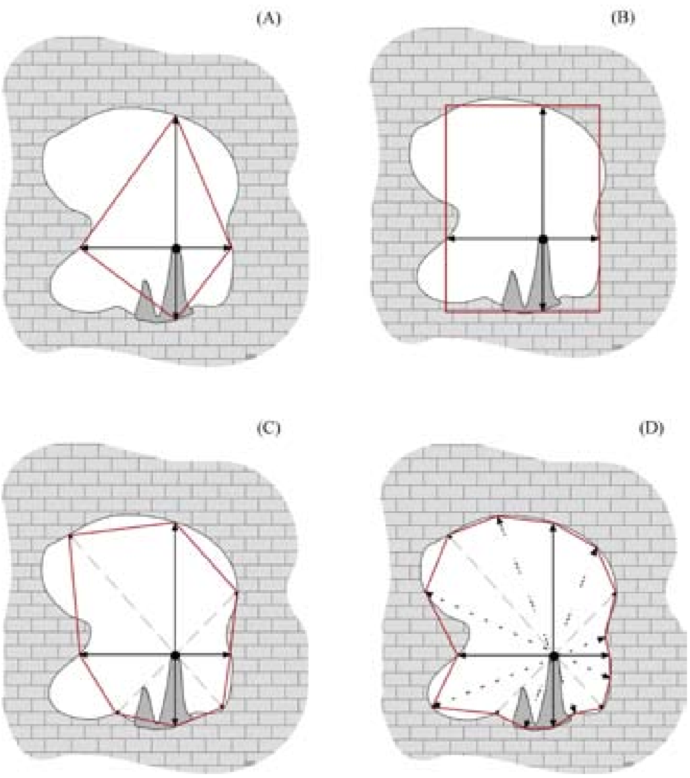

Figure 1. Determination of cave passage cross-sections. In standard surveying methods the distances from the survey station (on top of stalagmite in this example) to the left-right-up-down (LRUD) passage boundaries are measured or estimated as shown by arrows (A, B). An approximation of passage shape may then be made as a polygon using LRUD locations as vertices for a quadrilateral (A), or as a rectangular bounding box using LRUD to establish lines that are either vertical or horizontal (B), or as an ellipse using the sums L+ R and U + D (not shown). If additional radial measurements are collected, an octahedron using eight points as vertices (C) or hexadecagon with sixteen radial points may be made (D). In each case cross-section area of the passage may be calculated using geometric formulae.

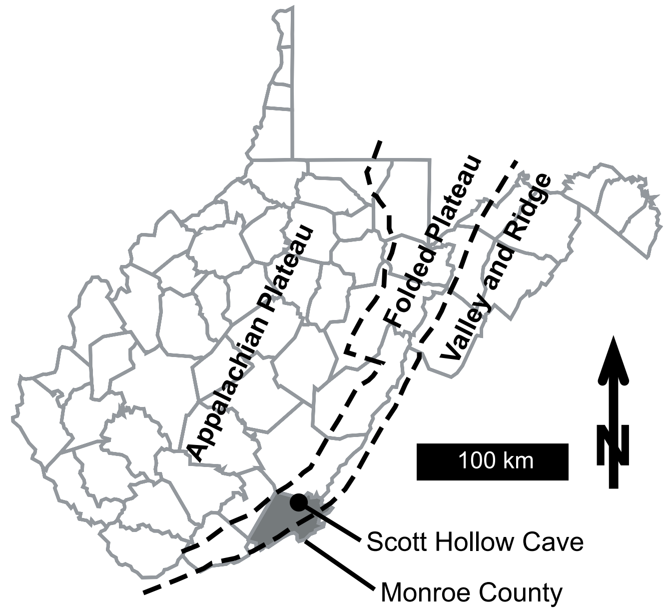

Figure 2. Location of Monroe County, West Virginia and Scott Hollow Cave, with respect to physiographic provinces. Province boundaries after Kulander and Dean (1986).

Methods

For this investigation, a series of radial cross-section surveys were conducted in several passages of Scott Hollow Cave. The locations of the 18 survey stations are shown in Fig. 3. These stations were chosen to encompass a variety of sizes and morphologies. At each location a specially designed survey data sheet was used. By convention, the survey was conducted facing into the cave. Between two and five surveyors were present at a given station. The names of each surveyor, location, passage type, date, time, and the survey number were recorded. Then, each surveyor present made a private estimation of LRUD (in feet) which was recorded for later comparison with the laser measurements.

After this, a laser rangefinder, the Leica Disto Classic5, was used to precisely measure radial distances to passage walls. This device measures distances up to 200 m and has an accuracy of ±1.5 mm. In order to conveniently make numerous precise measurements at each station, a transit head was modified from horizontal rotation to vertical rotation and mounted on a tripod to secure the rangefinder (Fig. 4). The head was aligned with the general trend of the cave passage at the survey station at the discretion of the operator. Intervals at 22.5º were marked on the transit head to allow for 16 radial measurements at equiangular steps (Fig. 1d). Distance measurements were taken in units of meters referenced to the rear of the instrument. Because the instrument measurement point was not centered on the rotation axis of the transit head, an offset of −0.106 m was programmed into the rangefinder to produce the correct measurement while surveying.

After the 16 measurements were obtained, a sketch of the cross-section was made indicating the station point and other detailed features of the cave passage. Various types of passages

Table 1. Methods by which passage cross-section dimensions can be used to represent a cave.

Method	Description
Plan View Maps	Passage width by scale drawing
	Passage height by notation (number in circle)
	Passage height, width, and shape by inclusion of selected passage cross-section drawings
Longitudinal Cross Sections	Passage height by scale drawing
Overall Cave Shape and Size	Perspective (or block) diagrams of cave (an artist uses the data to draw by hand an illustration of the cave)
	Physical (usually clay) models of cave (sculptor uses data to create representation of cave)
	Three-dimensional digital models (cross sections at each station are modeled from LRUD data, and then passage dimensions between stations are interpolated to construct a tube or prism approximating the passage shape)

were measured, including large and small trunk conduits, small keyhole-shaped canyons, and very large canyons. Rooms, which present special morphological problems for surveying, were excluded from consideration.

Measurements from each survey station were used to construct five geometric representations for each passage cross-section shape. By simply joining the ends of four, eight, and 16 measurement vectors, quadrilateral, octagon, and hexadecagon representations were made (see Fig. 1a, c, d). Polygons so constructed using vectors radiating at equal angles from a central point have been informally called radar polygons. Within this study these three polygon types are geometrically irregular in that they do not contain equal angles between their sides, though. Additionally, two other geometric representations were made using the four LRUD measurements to create rectangles and ellipses. In the former case this was done by using the L+R and U+D sums as values for the sides of a rectangle. In the latter case these values served as the axes of the ellipse.

Measurements and estimations from each survey station were entered into Excel data sheets to calculate the areas of each cross-section, and to allow for various comparisons. Areas for the quadrilateral, octagon, and hexadecagon were calculated by summing the areas of triangles defined by the measurement vectors. For example, in the case of the hexadecagon 16 triangles are present (Fig. 1d). The area of each triangle was found by taking one half the sine of the known angle (22.5º) times the length of the two known sides (adjacent radial measurements). Additionally, areas for rectangular and ellipse representations (using measured LRUD) were calculated. Areas (for quadrilateral, rectangle, and ellipse representations) based on estimations of LRUD were then compared to areas calculated from measured values in order to evaluate potential error that occurred from estimating the LRUD.

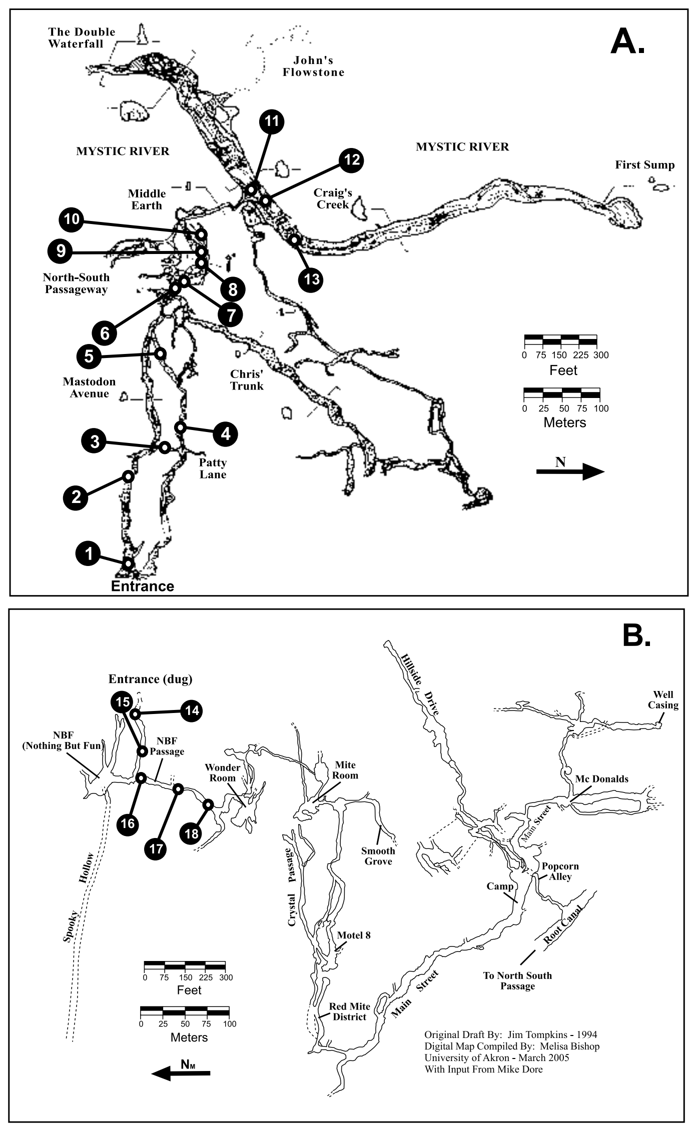

Figure 3. Maps of selected portions of Scott Hollow Cave showing locations of radial survey stations. Base maps courtesy Mike Dore.

Results and Analysis

Representation of Morphology

Figure 5 shows cross sections of all of the stations at the same scale, each represented with several different geometric approximations. The hexadecagon (column A) uses the greatest number of measurements (16) resulting in the most detailed, and hence most realistic, shape. This is apparent, for example, at stations 7, 11, 12 and 13. However, in several cases shape does not change substantially with the use of more radial points. Because of their diverse genesis and geologic controls, conduit cross sections have varying shapes and complexities (see examples in White, 1988). There are also many factors (irregularity, placement of survey station, *etc.*) which affect the number of points which are required to make a suitable representation. These factors are beyond the consideration of the present paper.

Depending upon the purpose of a survey, a reasonable approximation of passage shape might be made with four or eight measurements. For example, if the desired outcome is a general representation of overall cave pattern, then detail beyond measured LRUD is unnecessary. This decision would need to be taken by a surveyor on a case-by-case basis. It is also possible that the surveyor might selectively choose points to measure based upon morphology at the station (rather than measuring at a set angle interval). Even a surveyor with minimal experience could make a useful selection of such points, by briefly examining the shape and complexity of the conduit. This would then reduce the overall amount of data collected, and reduce survey time. The structure of the resulting dataset, however, would be irregular, producing complexity in later data analysis and representation.

Columns D and E of Figure 5 show two other geometric representations, which are simplifications of the passage that would typically be used in a digital cave model. Such generalizations allow for ease in data processing and in rendering of images. For the cases shown in Figure 5, there is significant variation from the detailed passage morphology (represented in column A). In a subsequent section, the effects of these approximations on cross-section area calculations will be evaluated.

Accuracy of surveyor estimations of LRUD

Five surveyors with experience levels varying from novice to expert made 104 estimates of LRUD distances in the course of this study. There was no obvious difference in estimate quality between surveyors, and the limited sample size does not permit detailed analysis of this relationship. Therefore, the estimates are analyzed *en masse*. Figure 6 compares surveyor estimates of LRUD to laser measured LRUD distances. The data would appear as a straight line with slope of one if all estimates were accurate. The best fit line shows that, on average, an overestimate of distance is made. The scatter of points shows that magnitude of errors increases in larger passages, as would be expected. However, percent errors (a comparison of measured to estimated values) do not increase (Fig. 7).

For this study, the average absolute error for length estimates (regardless of positive or negative sign) was 27% of the laser measured value. Underestimates are made along with overestimates, however, so these offset each other if signed (non-absolute) values are averaged. This results in a 17% (positive) overall average error for distance estimates. This is a considerable error compared to those usually found for length, azimuth, and inclination measurements in a cave survey, which are typically less than 1%. This error could then propagate to the calculation of passage cross-section areas, and overall karst porosity.

Figure 4. Equipment used for precise radial surveys. Device shown in use within Scott Hollow Cave (A). Disto laser measurement device by Leica is mounted on a tripod using a modified transit head (B, C). Arrangement allows both for leveling of device, and rotation through full 360 degrees of measurement.

Variability of cross section area due to estimation of LRUD

Passage cross-section area, which is of interest for reasons mentioned in the introduction, was calculated with radial length values. Using the LRUD estimates or measurements, several alternative geometric representations of the cross section may be made. The simplest is a quadrilateral constructed by joining the endpoints of the LRUD (Fig. 5, column C), and this was employed by us to appraise variability of calculated cross-section area caused by errors in LRUD length estimates (Fig. 8). As with the LRUD length errors, errors in calculated area tend to increase with passage size, and occur both as positive and negative discrepancies. However, the average error in calculated area was 57%, as opposed to the 27% error value for length measurements. This increase is because length measurements are multiplied to find area, producing a compound error.

Comparison of area based on alternative geometric representations using LRUD

As an alternative to the quadrilateral representation discussed in the preceding section, the left-right (LR) and up-down (UD) segments may be summed, and these values were used to construct rectangular or elliptical generalizations (Fig. 5, columns D and E, respectively). Commercial cave mapping

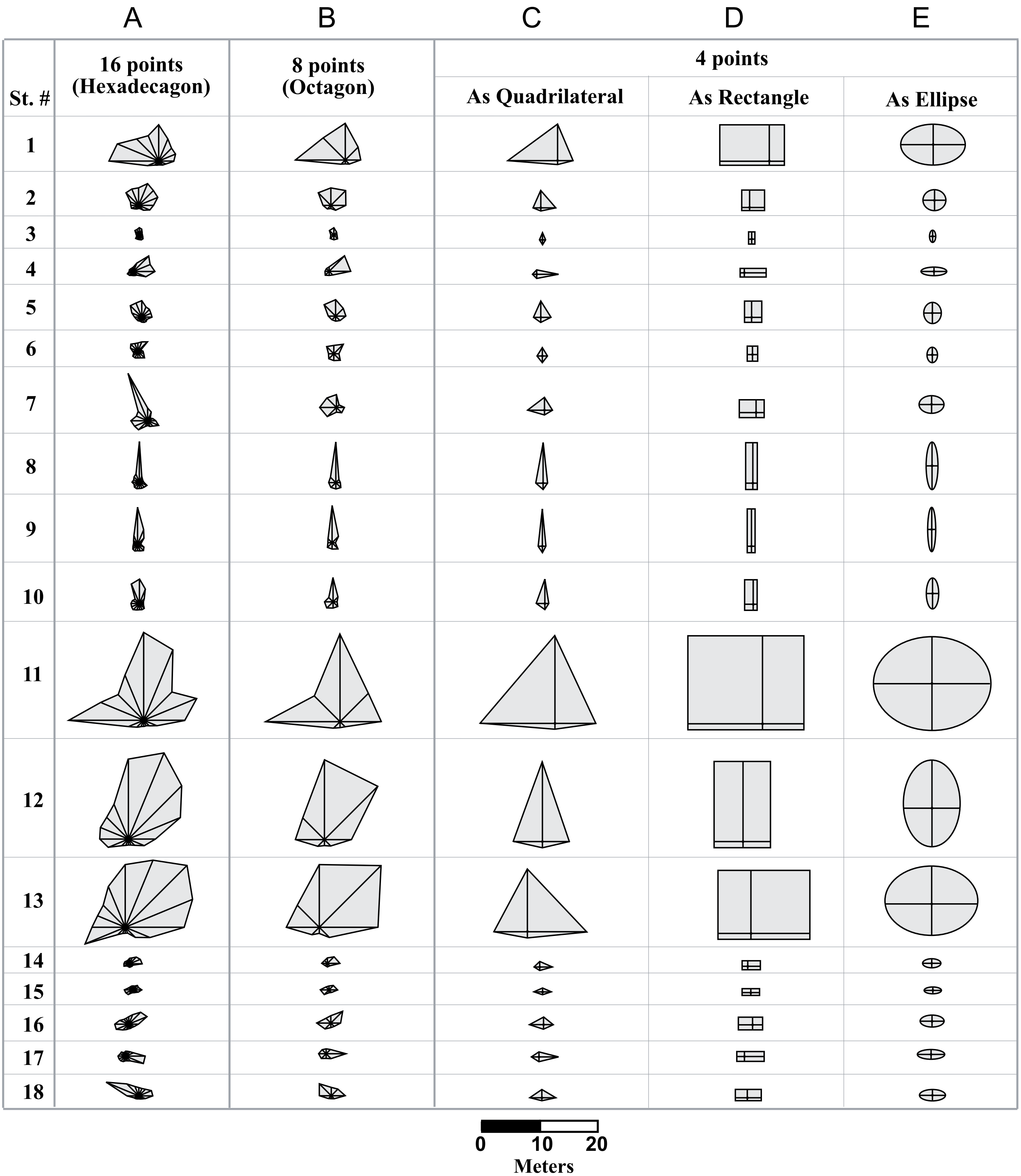

Figure 5. Graphical representation of surveyed cross sections for all stations. Thin black lines indicate up, down, left, right, or other radial distance measurements from survey station

software (*e.g.*, Compass, Walls, WinKarst) as well as advanced geometric visualization software (*e.g.*, EVS, GoCad, ArcGIS), may be employed to make such renderings. Depending upon the morphology of the conduit, different generalizations may provide a better or worse fit. For example, phreatic conduits having smooth perimeters would be most accurately modeled using the ellipse. To evaluate the aptness of the quadrilateral, rectangle, and ellipse generalizations, these areas were compared to those of the (non-regular) hexadecagon for each of the 18 stations (Fig. 9). Within the scope of this investigation, we consider that the hexadecagon provides the definitive passage cross section. The quadrilateral gave the least accurate value of cross-section area, underestimating in every case but two, with a range of −64% to +12% and a median error of −45%. The ellipse and rectangle provide more suitable values, with scatter on both sides of the hexadecagon value, and median errors of −11% and +10% respectively.

Discussion and Conclusions

The data collected for this study allow evaluation of two sources of potential error associated with standard techniques for determination of cave passage cross sections. These errors arise from (a) estimation, rather than measurement, of LRUD distances, and (b) the insufficiency of four orthogonal lengths to define complex shapes.

For the passages that were examined, morphology of the conduits was not well-represented using only LRUDs (Fig. 5). This demonstrates that for applications where it is important to show passage shape, it is essential that the surveyor either provide a hand-drawn cross section, and/or make additional radial measurements beyond the four standard LRUDs. Simple canyons or tubes can be represented with solely four measurements, but where complexity exists due to breakdown, passage intersections, or other factors, increasing the number of measurements makes a noticeable difference in representation of morphology.

With regard to the practice of estimating rather than measuring LRUDs, it was found that estimates made by surveyors are grossly accurate, but poor for analytical use, and far below typical survey standards for at least 1% accuracy. An average overestimate of 27% was found for all distance determinations. The magnitude of errors increased with larger passage sizes, but the percent errors did not increase. Length errors compound, however, when areas are calculated. This results in an average cross-section area error of 57% (as quadrilateral), which is problematic for such things as calculation of fluid storage volumes or paleodischarges.

Finally, a comparison of the use of different geometries to calculate cross-section area shows that representation as a quadrilateral results in a median overestimation of about 45%. Alternatively, only 10% error (+ or −) is associated with representing the conduit as a rectangle or ellipse. Passage shape obviously plays a role in how good the area representation is; simpler cross sections can be more accurately represented by a smaller number of measurements. Because conduit shape results from such factors as lithology, structure, and hydrologic history, it follows that certain caves will be more accurately represented by LRUD only, along with simplistic geometries. It might be possible in future work to quantify this effect by studying errors present in passages of different rock types, hydrologic origin, *etc.*

The data and analyses presented above illustrate the variety of factors that can affect the measurement and representation of cross-section data for cave passages. For the majority of cave

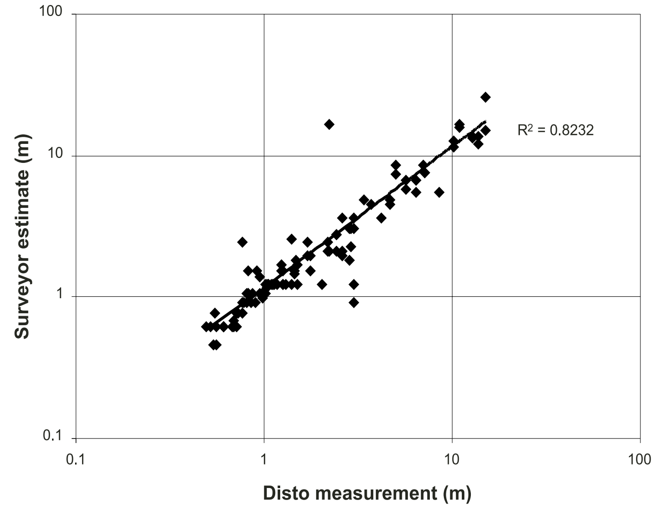

Figure 6. Comparison of LRUD distances as measured by Disto (x-axis) to surveyor estimates (y-axis). One hundred four estimates made by five different surveyors are included. Linear regression (solid line) using all points is given.

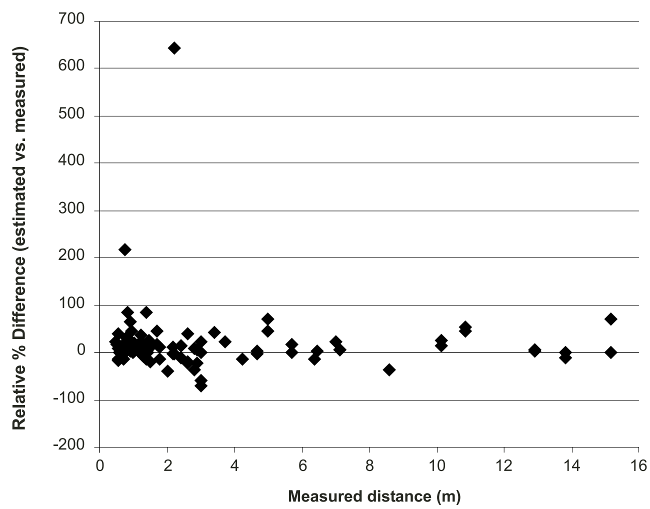

Figure 7. Relationship between measured LRUD distances and percent error in surveyor estimations of same. One hundred four comparisons are given. Seventy-three of the estimates exceeded the measured value (by an average of 32%). Thirty-one of the estimates were less than the measured value (by an average of 16%). Therefore, in this analysis, surveyors were about twice as likely to overestimate LRUD distances as to underestimate them. In addition, overestimates were worse (2×) in terms of percentage error. Surprisingly, there is no relation between the measured distance and the percent error of estimates.

maps, which are made for the purposes of navigation and for documenting exploration, estimation of LRUD probably provides a suitable estimate of true passage dimensions. With only these four measurements, the ellipse and rectangle provide the most accurate passage shape and area representations. Where more accurate measures of passage shape and area are required, the use of 16 radial measurements can be employed. This requires additional equipment and time, but may be justified where such accuracy is needed for scientific purposes. Nevertheless, there are many other limitations to the accurate representation of karst porosity. These particularly include rooms, which are not amenable to accurate description using the line and LRUD paradigm of most surveying approaches.

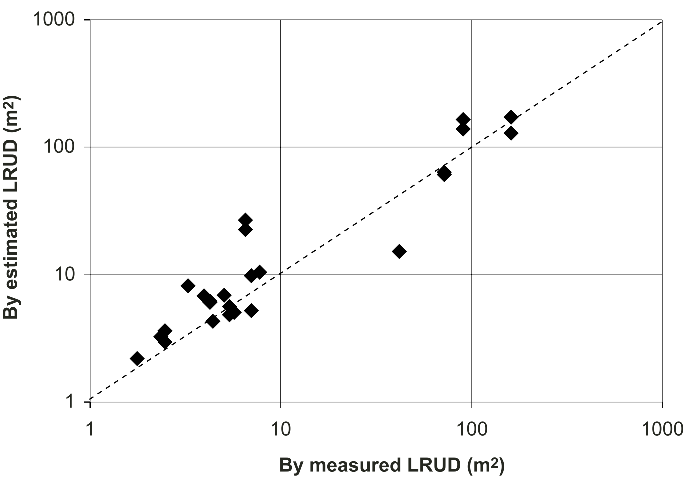

Figure 8. Comparison of conduit cross-section areas (m^2) calculated using measured vs. estimated LRUD values (as quadrilateral). Log-log scale is used in order to facilitate viewing of data. Dashed line illustrates the theoretical relationship that would exist if all LRUD estimates were accurate.

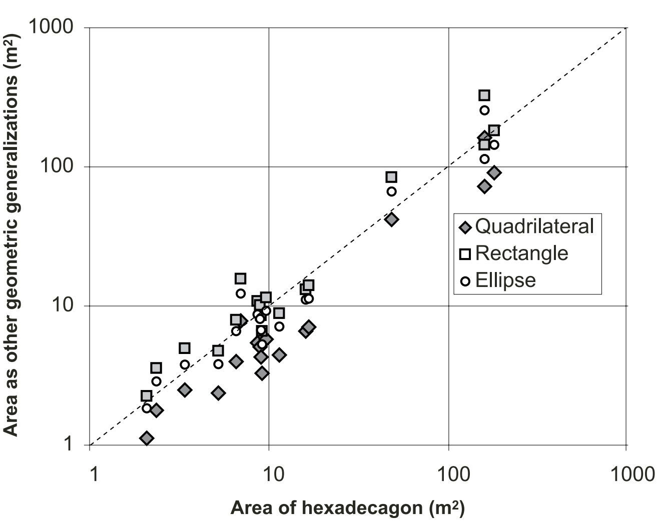

Figure 9. Comparison of passage cross-section areas (m^2) based on geometric generalizations (quadrilateral, rectangle, and ellipse) of measured LRUD values to areas based on 16 radial measurements (hexadecagon). The hexadecagon is the most accurate. The quadrilateral provides the least appropriate area, almost always under-representing the value. Log-log scale is used in order to facilitate viewing of data.

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